Note Title 3/7/2019

$$\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + 3y(t) = 4\frac{dx(t)}{dt} + 5x(t)$$

$$(jw)^2 Y(w) + 2(jw)Y(w) + 3Y(w) = 4(jw)X(w) + 5X(w)$$

$$-w^2 Y(w) + 2jw\,Y(w) + 3Y(w) = 4jw\,X(w) + 5X(w)$$

$$Y(w)\left[(3-w^2) + 2jw\right] = X(w)\left\{5 + 4jw\right\}$$

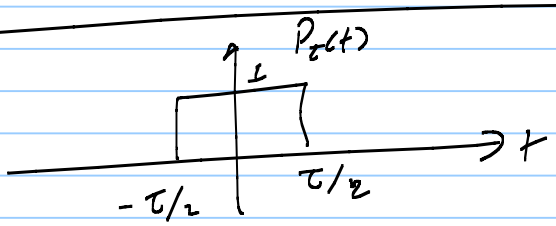
$$\frac{Y(w)}{X(w)} = \frac{5+j4w}{3-w^2+j2w}$$ Transfer function

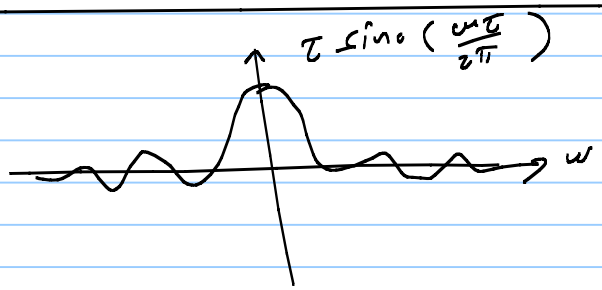
---

$x(t) \longrightarrow$ [Syst.] $\longrightarrow y(t)$

$\delta(t) \longrightarrow$ [Syst.] $\longrightarrow h(t)$

$$y(t) = x(t) * h(t)$$





$$x(t) \longleftrightarrow X(w)$$

$$P_\tau(t) \longleftrightarrow \tau\, sinc\left(\frac{w\tau}{2\pi}\right)$$

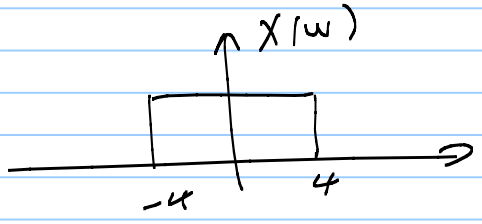
$$P_\tau(t) \leftrightarrow \tau \, \text{sinc}\left(\frac{\omega\tau}{2\pi}\right)$$

Even function

$$\tau \, \text{sinc}\left(\frac{t\tau}{2\pi}\right) \leftrightarrow 2\pi \, P_\tau(-\omega)$$

$$\leftrightarrow 2\pi \, P_\tau(\omega)$$

$$\frac{\tau}{2\pi} \, \text{sinc}\left(\frac{t\tau}{2\pi}\right) \leftrightarrow P_\tau(\omega)$$



find $x(t)$

$$\therefore x(t) = \frac{8}{2\pi} \, \text{sinc}\left(\frac{8t}{2\pi}\right)$$

$$X(\omega) = \frac{1}{j}\left[\text{sinc}\left(\frac{2\omega}{\pi} - \frac{1}{2}\right) - \text{sinc}\left(\frac{2\omega}{\pi} + \frac{1}{2}\right)\right]$$

$$= \frac{2}{2}(-j)\left[\quad\quad\right]$$

$$= -2\left(\frac{j}{2}\right)\left[\text{sinc}\left(\frac{4\omega}{2\pi} - \frac{1}{2}\right) - \text{sinc}\left(\frac{4\omega}{2\pi} + \frac{1}{2}\right)\right]$$

$$= 2\left(\frac{j}{2}\right)\left[\text{sinc}\left(\frac{4\omega}{2\pi} + \frac{1}{2}\right) - \text{sinc}\left(\frac{4\omega}{2\pi} - \frac{1}{2}\right)\right]$$

$$= 2\left(\frac{j}{2}\right)\left[\text{sinc}\left(\frac{4\omega+\pi}{2\pi}\right) - \text{sinc}\left(\frac{4\omega-\pi}{2\pi}\right)\right]$$

$$= 2\left(\frac{1}{2}\right)\left\{\frac{4}{4}\text{sinc}\left(\frac{4(\omega+\frac{\pi}{4})}{2\pi}\right) - \frac{4}{4}\text{sinc}\left(\frac{4(\omega-\frac{\pi}{4})}{2\pi}\right)\right\}$$

$$= \frac{1}{2}\left(\frac{1}{2}\right)\left\{4\,\text{sinc}\left(\frac{4(\omega+\frac{\pi}{4})}{2\pi}\right) - 4\,\text{sinc}\left(\frac{4(\omega-\frac{\pi}{4})}{2\pi}\right)\right\}$$

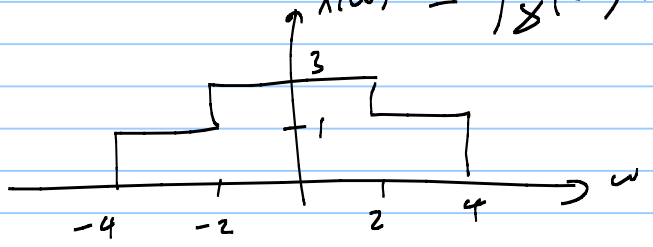
$$y(t) = \frac{1}{2} P_4(t)\sin\left(\frac{\pi}{4}t\right)$$

$$P_4(t) \leftrightarrow 4\,\text{sinc}\left(\frac{4\omega}{2\pi}\right)$$

$$\frac{1}{2} P_4(t)\sin\left(\frac{\pi}{4}t\right)$$

$$\leftrightarrow \frac{1}{2}\left[\frac{1}{2}\left(4\,\text{sinc}\left(\frac{4(\omega+\frac{\pi}{4})}{2\pi}\right) - 4\,\text{sinc}\left(\frac{4(\omega-\frac{\pi}{4})}{2\pi}\right)\right)\right.$$

$$X(\omega) = P_8(\omega) + 2P_4(\omega)$$



find $x(t)$

$$P_8(t) \leftrightarrow 8\,\text{sinc}\left(\frac{8\omega}{2\pi}\right)$$

$$8\,\text{sinc}\left(\frac{8t}{2\pi}\right) \leftrightarrow 2\pi P_8(\omega)$$

$$\frac{4}{\pi}\,\text{sinc}\left(\frac{4t}{\pi}\right) \leftrightarrow P_8(\omega)$$

$$2P_4(t) \leftrightarrow 8\,\text{sinc}\left(\frac{4\omega}{2\pi}\right)$$

$$8\,\text{sinc}\left(\frac{4t}{2\pi}\right) \leftrightarrow 2\pi(2P_4(\omega))$$

$$\frac{4}{\pi}\,\text{sinc}\left(\frac{2t}{\pi}\right) \leftrightarrow 2P_4(\omega)$$

$$x(t) = \frac{4}{\pi}\,\mathrm{sinc}\left(\frac{4t}{\pi}\right) + \frac{4}{\pi}\,\mathrm{sinc}\left(\frac{2t}{\pi}\right)$$